KINETIC MODEL OF A FLUIDIZED LAYER

(KINETICHESKAIA MODEL' KIPIASHCHEGO SLOIA)

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Theoretical methods of analysis of phenomena occurring in chemical apparatus with a 'fluidized layer' are of great interest at present, primarily in connection with the wide application of such equipment in industrial chemical technology.

The complexity and multistage nature of processes taking place in the fluidized layer leave little hope for a complete description of these processes by means of some single method. However, a characteristic feature of almost all processes taking place in the fluidized layer is their strong dependence on the nature of mechanical motion of solid particles which form the layer.

A large number of papers are devoted to both, the theoretical and experimental investigation of processes which take place in the fluidized layer. Nevertheless, at the present time a theoretical model has not yet been created which would describe in sufficient detail the characteristics of mechanical motion of solid particles forming the layer. Namely, in certain models used at present the nature of the sharp transition into the pseudo-fluidized state is not developed, distribution of particles with respect to velocities and the relationship between particles velocity distribution and gas flow parameters are not determined, etc.

In this paper the simplest kinetic model of a fluidized layer is presented based on the idea that the solid particles in the layer can be represented by an aggregate of elastic spheres, while their interaction with the suspending gas flow leads to diffusion of the point representing the particle in its velocity space.

1. Kinetic equation. We denote by $f(\mathbf{x}, \mathbf{u}, t)$ the distribution function of a number of particles such that the number of particles in the volume $[x_i, x_i + dx_i]$ possessing velocities in the interval $[u_i, u_i + du_i]$, is equal to $f(\mathbf{x}, \mathbf{u}, t)\, d\mathbf{x} d\mathbf{u}$, where $d\mathbf{x} = dx_1 dx_2 dx_3$ and $d\mathbf{u} = du_1 du_2 du_3$. For the purpose of simplicity it is assumed that the solid particles represent elastic spheres of equal size.

A change in the number of particles in the volume $d\mathbf{x}d\mathbf{u}$ is a result of convective transfer accorss the boundaries of the volume $d\mathbf{x}d\mathbf{u}$ and of mutual collisions of particles in the volume $d\mathbf{x}$. Then, taking into account that the change of particle velocity occurs, generally speaking, under the influence of forces which depend on the velocity of their motion and repeating almost verbatim the derivation procedure of the fundamental equation in the kinetic theory of gases [1 and 2], we obtain

$$\frac{\partial f}{\partial t} + \sum_{i=3}^{3} u_i \frac{\partial f}{\partial x_i} = - \sum_{i=1}^{3} \frac{\partial}{\partial u_i} \left[\frac{F_i}{m} f \right] + C(ff_1) \tag{1.1}$$

Here m is the mass of the individual particle, $C(ff_1)$ is the rate of change in the number of particles in the volume $d\mathbf{x}d\mathbf{u}$ due to their mutual collisions with particles in the volume $d\mathbf{x}$, F_i is the force acting on an individual particle.

Each particle in the volume $d\mathbf{x}d\mathbf{u}$ is acted upon by the force of gravity equal to $m\mathbf{g}$, where $\mathbf{g}$ is the acceleration due to gravity, and the force from the direction of the suspending stream. The latter force can be represented in the form of three terms. The first term results from the regular component of the suspending gas under the assumption that the distribution of particles is orderly and that their velocities are equal to each other. The two other terms represent the irregular component of the suspending flow and the deviation of mutual distribution and of velocities of particles from the ordered state. If the assumption is made that deviations from the average value of this force assume statistically independent values in non-overlapping time intervals, then, in analogy to Brownian motion [3 and 4] we obtain for this force

$$\mathbf{F}_2 = \varphi_2 (|\mathbf{u} - \mathbf{w}|)(\mathbf{w} - \mathbf{u}) - D\nabla \ln f$$

Therefore the full force acting on each particle is

$$\mathbf{F} = m\mathbf{g} + \varphi_1 (|\mathbf{q} - \mathbf{w}|)(\mathbf{q} - \mathbf{w}) + \varphi_2 (|\mathbf{w} - \mathbf{u}|)(\mathbf{w} - \mathbf{u}) - D\nabla \ln f$$
$$\mathbf{w} = \int \mathbf{u} f(\mathbf{x}, \mathbf{u}, t)\, d\mathbf{u} \tag{1.2}$$

Here $\mathbf{w}$ is the average velocity of particles, $\mathbf{q}$ is the average hydrodynamic velocity of the suspending flow; φ_1, and φ_2 are functions the form of which depends on the particle size, their shape, the density and viscosity of the suspending flow, etc; D is the diffusion coefficient in the velocity space of an individual particle. The relationship of the quantity D with other characteristics of motion will be discussed in the next paragraph.

Substituting (1.2) into (1.1) we obtain the kinetic equation

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3} \left\{ u_i \frac{\partial f}{\partial x_i} + \left[g_i + \frac{\varphi_1}{m}(q_i - w_i) \right] \frac{\partial f}{\partial u_i} \right\} =$$
$$= C(ff_1) + \sum_{i=1}^{3} \frac{\partial}{\partial u_i} \left[\frac{\varphi_2}{m} (u_i - w_i) f + \frac{D}{m} \frac{\partial f}{\partial u_i} \right] \tag{1.3}$$

2. Solution of kinetic equation for small velocities of random motion. Suspension of a layer consisting of sufficiently friable and heavy particles by the stream of gas takes place at sufficiently high velocities of motion of gas relative to the particles. The magnitude of force acting on each particle in the gas stream depends not only on the velocity of the surrounding flow but also on the presence of neighboring particles. The latter is related to the mutual interaction of perturbations which are introduced by particles into the surrounding gas stream. The exact expression for φ_1 in (1.2) is not known, but various semi-empirical relationships are proposed for the determination of the magnitude of force acted by the flow upon each particle [5]. These relationships can be used for detailed quantitative computations. For the analysis of qualitative features of the process, φ_1 can be assumed to be

$$\varphi_1 = \lambda_0 \mid \mathbf{q} - \mathbf{w} \mid^{\beta-1} \qquad (\beta \geqslant 1) \tag{2.1}$$

Here λ_0 is a constant which depends on the shape and size of particles, on the density of particles, on the suspending flow and its viscosity etc; β is a dimensionless quantity.

If the velocities of the random motion of particles with respect to their average velocity are not large, then with sufficient degree of accuracy it may be assumed, that

$$\varphi_2 = \lambda = \text{const} \tag{2.2}$$

Let the particles be located above some horizontal boundary plane through which the vertical suspending gas flow enters the system. The axis x_1 is oriented vertically upwards, while the axes x_2 and x_3 are located in the boundary plane.

We shall examine the case of stationary state of the suspended layer. Taking into account (2.1) and (2.2), we obtain

$$\sum_{i=1}^{3} u_i \frac{\partial f}{\partial x_i} + \left[\frac{\lambda_0}{m} q^\beta - g\right] \frac{\partial f}{\partial u_1} = C(ff_1) + \sum_{i=1}^{3} \frac{\partial}{\partial u_i}\left[\frac{\lambda}{m} u_i f + \frac{D}{m}\frac{\partial f}{\partial u_i}\right] \tag{2.3}$$

We shall look for the solution in the form

$$f(\mathbf{x}, \mathbf{u}) = \Phi(x_1)\, \varphi(\mathbf{u}) \tag{2.4}$$

We note that the term in (2.3) which represents the collisions between the particles becomes equal to zero on the substitution of a function of the form

$$\varphi(u) = a\, e^{-\gamma v^2}$$

for arbitrary a and γ. If we write $\gamma = \lambda/2D$, then substituting into the right hand side of (2.3) the function

$$f(x, u) = \Phi(x_1) \exp\frac{-\lambda u^2}{2D} \tag{2.5}$$

we obtain zero, and the condition that the left-hand part of (2.5) becomes equal to zero, leads to the equation for the determination of $\Phi(x_1)$

$$\frac{d\Phi}{dx_1} = \frac{\lambda}{D}\left[\frac{\lambda_0}{m} q^\beta (x_1) - g\right]\Phi \tag{2.6}$$

By definition, the number of particles per unit volume is

$$n(x_1) = \int f(\mathbf{x}, \mathbf{u})\, d\mathbf{u} = \left(\frac{2\pi D}{\lambda}\right)^{3/2} \Phi(x_1)$$

Hence

$$f(x, u) = n(x_1)\left(\frac{\lambda}{2\pi D}\right)^{3/2} \exp\frac{-\lambda u^2}{2D} \tag{2.7}$$

The quantity q depends on x_1 and is related to n in some way. From the condition that the mass of gas flowing through any plane perpendicular to axis x_1 is constant, we obtain

$$q(x_1)\, \varepsilon(x_1) = Q = \text{const} \tag{2.8}$$

where $\varepsilon(x_1)$ is the relative volume occupied by gas. If

$$N_0 = 6 \,/\, \pi d^3 \tag{2.9}$$

where d is the diameter of the particle, then

$$\varepsilon\,(x_1) = 1 - n\,(x_1) \,/\, N_0 \tag{2.10}$$

Let us introduce the following dimensionless quantities

$$x_1 = h_0 z, \qquad R = Q\,(\lambda_0 \,/\, mg)^{1/\beta} \tag{2.11}$$

where h_0 is the initial thickness of the layer of solid particles. Then

$$\frac{d\varepsilon}{dz} = \frac{\lambda g h_0}{D}\left(1 - \frac{R^\beta}{\varepsilon^\beta}\right)(1 - \varepsilon) \tag{2.12}$$

The condition permitting a unique solution of equation (2.12) can be obtained from the requirement of conservation of the number of particles in the fluidized layer. In the absence of the suspending flow, the number of particles to be found in a cylinder with a base area equal to unity will be $N_1 h_0$, where N_1 is the number of particles per unit volume under the condition of dense packing of these particles. It is obvious that $N_1 < N_0$. Since the total number of particles in the cylinder should also be conserved in the presence of the suspending flow, we will have

$$\int\limits_0^\infty [1 - \varepsilon\,(z)]\,dz = \frac{N_1}{N_0} \tag{2.13}$$

Before investigating the equation (2.12) with the condition (2.13), it is necessary to analyze more closely parameters which determine the solution of kinetic equation in the form (2.4). This solution depends on the following parameters:

$$\frac{\lambda_0 Q^\beta}{mg}, \quad \frac{N_1}{N_0}, \quad \frac{\lambda}{D}, \quad \frac{\lambda g h_0}{D}, \quad \beta \tag{2.14}$$

First of them characterizes the intensity of fluidization of the particle by the flow of gas. The value of the ratio N_1/N_0 is known if the particle diameter is known and it characterizes the minimum porosity of the layer. The quantities λ_0, λ and β are determined from the hydrodynamic properties of particles.

The diffusion coefficient in the velocity space can be easily related to other characteristics of the process which permit a quite descriptive physical interpretation. The average kinetic energy of motion of an individual particle does not depend on x_1, and this can be shown analogously to the way it is done in the classical kinetic theory of gases [6].

After some simple computations, we obtain

$$\langle u^2 \rangle = 3D \,/\, \lambda \tag{2.15}$$

If λ and the RMS velocity of the random motion are known, then D can be easily found. We note that relationship (2.15) can be given another form. In fact, the average rate of kinetic energy dissipation of the individual particle is

$$\langle dE \,/\, dt \rangle = \langle \lambda\,(\mathbf{u} \cdot \mathbf{u}) \rangle = \lambda\,\langle \mathbf{u}^2 \rangle = 3D \tag{2.16}$$

In case when the quantity $\langle dE/dt \rangle$ is known or determinable by of direct measurements, the subsequent determination of D does not present any difficulties.

The introduction of parameter D into the kinetic equation is related to the irregularity of the true velocity field of the suspending flow. As a first approximation it may be assumed that the intensity of variation of average characteristics of the suspending flow depends only on Q. The forces acting on a particle and producing irregular changes in its velocity naturally will depend on λ and α and also on the density of the suspending flow, its viscosity etc. If parameters Q, λ and d are taken as the fundamental dimensional characteristics of these forces then

$$D = \lambda Q^2 D_0 \quad (2.17)$$

where, in accordance with what was said above, D_0 will be a slowly varying function of parameters enumerated above. In the first approximation D_0 may apparently be assumed to be a constant.

Utilizing (2.17) we finally obtain

$$R^2 \frac{d\varepsilon}{dz} = \frac{gh_0}{D_0}\left(\frac{\lambda_0}{mg}\right)^{2/\beta}\left(1 - \frac{R^\beta}{\varepsilon^\beta}\right)(1-\varepsilon) \quad (2.18)$$

Solution of Equation (2.18) always exists under the condition (2.13) and is unique, if $0 \leqslant R \leqslant 1$.

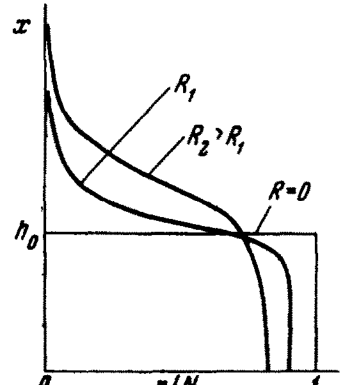

FIG. 1

Analysis of the solution can be carried out by the usual, well-known methods. Qualitative form of the solution is shown on fig. 1. For $R = 1$ we have $n(x_1) = 0$ for all x_1. This result appears completely natural, since for $R = 1$, each particle can be in equilibrium in the suspending flow, unaffected by the presence of other particles. If $R < 1$, then the solution of (2.18) always has a point of inflection, the coordinate x_1^* of which approaches h_0 when $R \to 0$, while dn/dx at the point of inflection tends to $-\infty$ when $R \to 0$.

The coordinate x_1^* of the inflection point for small R, can be taken in the first approximation, as the effective thickness of the suspending layer.

The solution constructed above is based on the assumption that translation of particles along the vertical is unrestricted. It is interesting to investigate the character of the solution of the kinetic equation in the gap between two parallel horizontal planes which restrict the possible vertical translation of particles. The distance between planes is designated by h_r, and $(h_r/h_0) = z_0 > 1$. The condition (2.13) is now rewritten in the form

$$\int_0^{z_0} (1 - \varepsilon(z))\, dz = \frac{N_1}{N_0} \quad (2.19)$$

Solution of (2.18) also exists for the condition (2.19) and appears to be unique. Its qualitative character is shown for various R on Fig. 2.

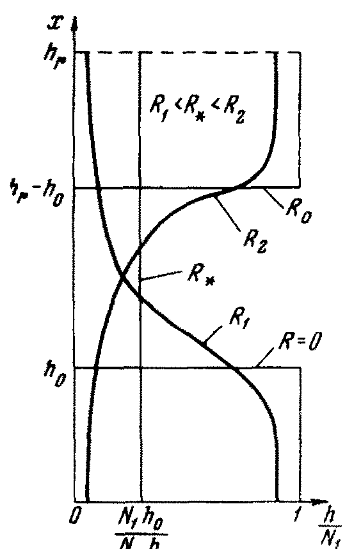

FIG. 2

We note that for some $Q = Q^*$ a solution $\varepsilon(z) = R^*$, exists, and

$$R^* = 1 - \frac{h_0}{h_r}\left(\frac{N_1}{N_0}\right) \tag{2.20}$$

From this

$$Q^* = \left(\frac{mg}{\lambda_0}\right)^{1/\beta}\left[1 - \frac{h_0}{h_r}\left(\frac{N_1}{N_0}\right)\right] \tag{2.21}$$

In this case the average number of particles in any vertical cross-section of the layer is constant, and the distribution function is

$$f(x, u) = \frac{N_1 h_0}{h_r}\left(\frac{\lambda}{2\pi D}\right)^{3/2} \exp\frac{-\lambda u^2}{2D} \tag{2.22}$$

3. Spatially homogeneous distribution for non-linear resistance law. We shall now examine the problem of finding the distribution function in the case when the velocities of the random motion of particles are sufficiently large and the linear, relationship (2.2) must be defined more accurately. We write

$$\varphi_2 = \lambda(1 + \alpha u) \tag{3.1}$$

where α is some positive constant, and in the following we shall limit ourselves to the analysis of a stationary, spatially homogeneous condition. The kinetic equation in this case is rewritten in the following way:

$$C(ff_1) + \sum_{i=1}^{3} \frac{\partial}{\partial u_i}\left[\frac{\lambda}{m}(1 + \alpha u)u_i f + \frac{D}{m}\frac{\partial f}{\partial u_i}\right] = 0 \tag{3.2}$$

In analogy to the Chapman-Enskog method [2] we shall seek the solution of (3.2) in the form

$$f = f^{(0)} + f^{(1)} + \ldots \tag{3.3}$$

and we shall limit ourselves to the first approximation. Substituting (3.3) into (3.2), we obtain for $f^{(0)}$ and $f^{(1)}$

$$C(f^{(0)} f_1^{(0)}) = 0 \tag{3.4}$$

$$C(f^{(0)} f_1^{(1)}) + C(f_1^{(0)} f^{(1)}) + \sum_{i=1}^{3} \frac{\partial}{\partial u_i}\left[\frac{\lambda}{m}(1 + \alpha u)u_i f^{(0)} + \frac{D}{m}\frac{\partial f^{(0)}}{\partial u_i}\right] \tag{3.5}$$

From (3.4) it follows in the general case, that

$$f^{(0)} = ae^{-\gamma u^2} \tag{3.6}$$

where a and γ are arbitrary positive constants.

We note that the properties of the collision integral impose certain restraints on the function of

$$\int \Psi^{(s)} \sum_{i=1}^{3} \frac{\partial}{\partial u_i}\left[\frac{\lambda}{m}(1 + \alpha u)u_i f + \frac{D}{m}\frac{\partial f}{\partial u_i}\right] d\mathbf{u} = 0 \qquad (s = 1, 2, 3) \tag{3.7}$$

Here $\psi^{(s)}$ are additive invariants of collisions [2]. For $\Psi^{(1)}=m$, $\Psi^2=mu$ Equation (3.7) is satisfied automatically if f and its partial derivatives with respect ot velocities tend to zero sufficiently rapidly when $u\to\infty$, and also by virtue of spatial isotropy of the state, in which no directions predominate in the phase space of the particle. Conversely, for $\Psi^{(3)}={}^1/_2\, mu^2$ condition (3.7) leads to the non-trivial relationship

$$3D = \langle dE \,/\, dt\rangle \tag{3.8}$$

(completely analogous to 2.16).

On substitution of (3.3) into (3.7), a series of conditions is obtained for the functions $f^{(0)}$, $f^{(1)}$, etc. Function $f^{(0)}$ is determined by these conditions with the accuracy to an arbitrary factor

$$f^{(0)} = a\exp\left[-(1+\xi)\frac{\lambda u^2}{2D}\right], \qquad \xi(1+\xi)^{1/2} = \frac{8\alpha}{3}\left(\frac{2D}{\pi\lambda}\right)^{1/2} \tag{3.9}$$

By virtue of triviality of (3.7) we can impose on $\Psi^{(1)}$ and $\Psi^{(2)}$ the condition

$$\int \Psi^{(1)} f^{(1)} d\mathbf{u} = \int \Psi^{(2)} f^{(1)} d\mathbf{u} = 0 \tag{3.10}$$

Then, from the normalising condition

$$N = \int f d\mathbf{u} = \int f^{(0)} d\mathbf{u} \tag{3.11}$$

we can determine d, and finally obtain for $f^{(0)}$

$$f^{(0)} = N\left[\frac{(1+\xi)\lambda}{2\pi D}\right]^{3/2}\exp\left[-(1+\xi)\frac{\lambda u^2}{2D}\right] \tag{3.12}$$

Let us introduce the dimensionless particle velocity

$$\mathbf{c} = \left[(1+\xi)\frac{\lambda}{2D}\right]^{1/2}\mathbf{u} \tag{3.13}$$

Now, Equation (3.5) can be written in terms of c_i, in the following form

$$S\int_0^{2\pi} d\varepsilon \int_0^{1/2\pi} d\psi \int dc_1\,[F^* + F_1^* - F - F_1]\,|c - c_1|\,e^{-c^2-c_1^2}\sin\psi\cos\psi = \\ = \xi(1+\xi)^{1/2}\sum_{i=1}^{3}\frac{\partial}{\partial c_i}\left[\left(\frac{3\sqrt{\pi}}{8}c - 1\right)c_i e^{-c^2}\right] \tag{3.14}$$

Here

$$f^{(1)} = f^{(0)}F, \quad S = \frac{Nd^2 m}{\pi\lambda}\left(\frac{2D}{\pi\lambda}\right)^{1/2} \tag{3.15}$$

and in agreement with the notation accepted in the kinetic theory of gases [2] the indices of F indicate the form of dependence of the argument on the variables of integration.

The general scalar solution of equation (3.14) can be represented in the form of a sum of the general solution of the homogeneous equation and the particular solution of the inhomogeneous equation

$$F = A_1 + A_3 c^2 + \sum_{i=1}^{3} A_2^{\,i} c_i + \Phi \tag{3.16}$$

Here A_1, A_2^i and A_3 are arbitrary constants and Φ is the particular solution of the inhomogeneous equation.

We note that the conditions of solvability of Equation (3.14) are satisfied as long as the conditions (3.7) are satisfied for the function $f^{(0)}$.

The particular solution of the inhomogeneous equation can be formed using the well known method in the kinetic theory of gases for representation of the solution in the form of a series in Sonin [2] polynomials. In the case under examination we write

$$F = \frac{\xi(1+\xi)^{1/2}}{S} = \frac{\xi(1+\xi)^{1/2}}{S}\sum_{r=0}^{\infty} a_r S_1^{(r)}(c^2) \tag{3.17}$$

Then

$$\int_0^{2\pi} d\varepsilon \int_0^{1/2\pi} d\psi \int d\mathbf{c}_1 |\mathbf{c}-\mathbf{c}_1| [\varphi^* + \varphi_1^* - \varphi - \varphi_1] e^{-c^2-c_1^2} \sin\psi\cos\psi = \\ = \left[S_1^{(0)}(c^2) - 2S_1^{(1)}(c^2) + \frac{3\sqrt{\pi}}{4} c S_1^{(1)}(c^2)\right] e^{-c^2} \tag{3.18}$$

Here $S_1^{(k)}(c^2)$ is the Sonin polynomial of order k.

Conditions (3.8) and (3.10) can be satisfied if in (3.16) we write $A_1 = A_2^i = A_3 = 0$. In fact, equality of A_2^i to zero necessarily follows from spatial isotropy of the state (it is also possible to convince oneself in the validity of this statement by means of direct calculations). Terms with A_1 and A_3 can be represented in the form of linear combination of $S_1^{(0)}$ and $S_1^{(1)}$ and included in a_0 and a_1 in the expansion (3.17). At the same time a_0 and a_1 must be determined from the conditions (3.8) and (3.10), since $S_1^{(0)}$ and $S_1^{(1)}$ are the eigenfunctions of the integral operator found in the left-hand part of (3.18) and are orthogonal with respect to the right-hand side of this equation.

Multiplying (3.18) by $S_1^{(k)}(c^2)$ and integrating over all c we shall obtain an infinite system of equations for determination of a_r $(r \geqslant 2)$

$$\sum_{r=2}^{\infty} A_{rk} a_r = B_k \qquad (k \geqslant 2) \tag{3.19}$$

$$A_{rk} = \int_0^{2\pi} d\varepsilon \int_0 d\psi \int d\mathbf{c}_1 |\mathbf{c}-\mathbf{c}_1| \sin\psi\cos\psi [S_1^{(r)}(c^{*2}) + S_1^{(r)}(c_1^{*2}) - \\ - S_1^{(r)}(c^2) - S_1^{(r)}(c_1^2)] S_1^{(k)} e^{-c^2-c_1^2} \qquad (r,\ k \geqslant 2) \tag{3.20}$$

$$B_k = \int [S_1^{(0)}(c^2) - 2S_1^{(1)}(c^2) + \frac{3\sqrt{\pi}}{4} c S_1^{(1)}(c^2)] e^{-c^2} S_1^{k}(c^2)\, d\mathbf{c} = \\ = 2\pi \sum_{p=0}^{k} (-1)^p \binom{1+k}{k-p} (2p-1) \qquad (k \geqslant 2) \tag{3.21}$$

Conditions (3.8) and (3.10) allow us to express a_0 and a_1 immediately in terms of $a_r (r \geqslant 2)$

$$a_0 = -\sum_{r=2}^{\infty} a_r \left[\frac{1}{4} - \frac{1}{64}\sum_{p=0}^{r} (-1)^p \binom{r+1}{r-1}\binom{2p+1}{p}\frac{(2p+3)(p+1)}{4^p}\right] \tag{3.22}$$

$$a_1 = -\sum_{r=2}^{\infty} a_r\left[\frac{3}{4} + \frac{1}{64}\sum_{p=0}^{r}(-1)^p\binom{r+1}{r-p}\binom{2p+1}{p}\frac{(2p+3)(p+1)}{4^p}\right]$$

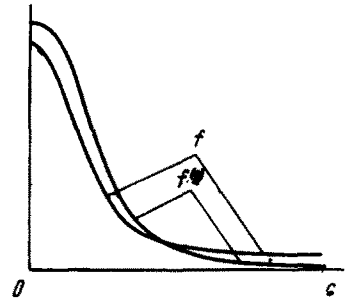

FIG. 3

Coefficients a_r $(r \geqslant 2)$ can be found with any degree of accuracy, by means of the method of successive approximations employed in the kinetic theory of gases. The corresponding procedure of the method of successive approximations consists in the replacement of the infinite series by a finite sum of first n members with subsequent solution of a system of n linear algebraic equations.

If we limit ourselves to a series with three terms, we obtain for the distribution function

$$f = f^{(0)}\,[1 + 2\pi\xi\,(1+\xi)^{1/2}\,S^{-1}\,(c^4 - 1.53c^2 - 1.13)] \qquad (3.23)$$

For small ξ the behavior of $f^{(0)}$ and f is shown qualitatively in fig. 3. Non-linearity in the resistance manifests itself in, the fact that the region of 'concentration' of the distribution function widens.

Finally we note that the constructed model of the fluidized layer leads to results which are in good qualitative agreement with experiments. In particular, for small velocities of suspending flow the proposed model leads to a sharply developed layer with almost constant number of particles per unit volume along the height within the boundaries of the layer. Also, the function f contains exhaustive information about average characteristics of mechanical motion of particles and permits, in principle to compute any average characteristics of processes taking place in the fluidized layer if the characteristics of elementary processes taking place on each separate particle and associated with its mechanical motion, are known.

As an example we shall examine the problem of computing the average velocity of reaction in a unit volume of the fluidized layer if the reaction proceeds only on the surface of particles and if the rate of the conversion process on each particle is determined only by the flow of reagent onto the particle surface. When no collision occurs the flow of the reagent onto the surface of the particle is determined by the velocity of motion of the particle, while during the collisions, it is determined by their relative velocity because of large velocity gradients between particles. Denoting by $q_1(u_1)$ and $q_2(u_1 - u_2)$ the flow of reagent onto the surface of a particle while it moves between the collisions and during the moments of collisions respectively, we obtain for the average velocity of reaction per unit volume of fluidized layer

$$q = \int q_1(\mathbf{u}_1)\,f(\mathbf{x},\mathbf{u}_1,t)\,d\mathbf{u}_1 +$$

$$+ d^2\int_0^{2\pi} d\varepsilon \int_0^{\frac{1}{2}\pi} d\psi \iint d\mathbf{u}_1\,d\mathbf{u}_2\,|\mathbf{u}_1 - \mathbf{u}_2|\,q_2(\mathbf{u}_1 - \mathbf{u}_2)\,f(\mathbf{x},\mathbf{u}_1,t) f(\mathbf{x},\mathbf{u}_2,t)\sin\psi\cos\psi$$

In this manner the computation is reduced to computation of quantities $q_1(u)$ and $q_2(u_1 - u_2)$ for the individual particle, i.e. essentially to a solution of hydrodynamic problems for which at present time the methods of solution are sufficiently well worked out.

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